Roll No. ....

94060

B. Sc. (Hons.) Mathematics
5th Semester Old/New Scheme
Examination – February, 2022

GROUPS AND RINGS

Paper : BHM-352

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. 9 (Section – V) is *compulsory*. All questions carry equal marks.

SECTION – I

1. (a) Prove that a necessary and sufficient condition for a non-empty finite subset H of group $(G, .)$ to be a subgroup is that H must be closed with respect to multiplication. 6

(b) State and prove Lagrange's theorem for finite groups. 6

2. (a) If G is the additive group of integers and H is the subgroup of G obtained on multiplying the elements of G by 4, find the index of H in G. 6

(b) Prove H is a normal subgroup of G iff $xHx^{-1} = H$ for all $x \in H$. 6

SECTION – II

3. (a) State and prove Fundamental theorem on Homomorphism of Groups. 6

(b) If G is a finite abelian group of order n and m is a positive integer such that $(m, n) = 1$ then show that $f : G \to G$ defined by $f(x) = x^m$ is anautomorphism. 6

4. (a) Let $Z(G)$ be the centre of a group G. If G/Z is cyclic, then prove that G is abelian. 6

(b) If $S = \{1, 2, 3, 4, 5, 6, 7\}$ and $f = (1, 3)$, $g = (2, 4, 6)$, show that $fog = gof$. 6

SECTION – III

5. (a) Show that centre of a ring R is a subring of R. 6

(b) Prove that every field is a principal ideal ring. 6

6. (a) Prove that every homomorphic image of a ring R is isomorphic to some quotient ring. 6

(b) An ideal S of a commutative ring R with unity is maximal iff R/S is a field. 6

SECTION – IV

7. (a) The ring of Gaussian integers is an Euclidean domain. 6

(b) Show that every non-zero prime ideal of a principal ideal domain is maximal. 6

8. (a) If F is a field, then $F[x]$ is a principal ideal ring. 6

(b) Show that if a is an irreducible element of a unique factorization domain R then a must be prime. 6

SECTION – V

(Compulsory Question)

9. (a) Find the principal ideal generated by 3, in the ring of integers. 12

(b) Show that the mapping $f : C \to C$ defined by $f(a + ib) = a - ib$ is a homomorphism.

(c) Prove that the product of two units $a, b \in R$ is also a unit of R.

(d) Let $G = \{1, 2, 3, 4\}$ be the group w.r.t. multiplication modulo 5. Find order of each element.

(e) Prove that every group of prime order is cyclic.

(f) Prove that identity mapping is the only inner automorphism for an abelian group.